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Cambridge Assessment International Education

Cambridge International AS & A Level

CANDIDATE NAME

CENTRE NUMBER

CANDIDATE NUMBER

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FURTHER MATHEMATICS **9231/42**

Paper 4 Further Probability & Statistics **October/November 2024**

1 hour 30 minutes

You must answer on the question paper.

You will need: List of formulae (MF19)

INSTRUCTIONS

- Answer **all** questions.
- Use a black or dark blue pen. You may use an HB pencil for any diagrams or graphs.
- Write your name, centre number and candidate number in the boxes at the top of the page.
- Write your answer to each question in the space provided.
- Do **not** use an erasable pen or correction fluid.
- Do **not** write on any bar codes.
- If additional space is needed, you should use the lined page at the end of this booklet; the question number or numbers must be clearly shown.
- You should use a calculator where appropriate.
- You must show all necessary working clearly; no marks will be given for unsupported answers from a calculator.
- Give non-exact numerical answers correct to 3 significant figures, or 1 decimal place for angles in degrees, unless a different level of accuracy is specified in the question.

INFORMATION

- The total mark for this paper is 50.
- The number of marks for each question or part question is shown in brackets [ ].

This document has **16** pages. Any blank pages are indicated.

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1 A scientist is investigating the lengths of the leaves of a certain type of plant. The scientist assumes that the lengths of the leaves of this type of plant are normally distributed. He measures the lengths, x cm, of the leaves of a random sample of 8 plants of this type. His results are as follows.

3.5 4.2 3.8 5.2 2.9 3.7 4.1 3.2

Find a 90% confidence interval for the population mean length of leaves of this type of plant. [4]

2 The random variable X has probability generating function $G_X(t)$ given by

$$G_X(t) = \frac{1}{5} + pt + qt^2,$$

where p and q are constants.

(a) Given that $E(X) = 1.1$, find the numerical value of $Var(X)$. [4]

The random variable Y has probability generating function $G_Y(t)$ given by

$$G_Y(t) = \frac{2}{3}t\left(1 + \frac{1}{2}t^2\right).$$

The random variable Z is the sum of independent observations of X and Y.

(b) Find the probability generating function of Z. [2]

(c) Find $P(Z > 2)$. [1]

(d) State the most probable value of Z. [1]

3 Rosie sows 5 seeds in each of 150 plant pots. The number of seeds that germinate is recorded for each pot. The results are summarised in the following table.

Number of seeds that germinate	0	1	2	3	4	5
Number of pots	12	40	43	35	16	4

Rosie suggests that the number of seeds that germinate follows the binomial distribution B(5, p).

(a) Use Rosie's results to show that $p = 0.42$. [1]

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(b) Carry out a goodness of fit test, at the 10% significance level, to test whether the distribution B(5, 0.42) is a good fit for the data. [9]

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4 The random variable X has probability density function f given by

$$f(x) = \begin{cases} \frac{1}{21}(x-1)^2 & 2 \leqslant x \leqslant 5, \\ 0 & \text{otherwise.} \end{cases}$$

(a) Find the cumulative distribution function of X. [3]

The random variable Y is defined by $Y = (X-1)^4$.

(b) Find the probability density function of Y. [3]

(c) Find the median value of Y. [2]

(d) Find $E(Y)$. [2]

5 Dev owns a small company which produces bottles of juice. He uses two machines, X and Y, to fill empty bottles with juice. Dev is investigating the volumes of juice in the bottles. He chooses a random sample of 35 bottles filled by machine X and a random sample of 60 bottles filled by machine Y. The volumes of juice, x and y respectively, measured in suitable units, are summarised by

$$\sum x = 30.8, \qquad \sum x^2 = 29.0, \qquad \sum y = 62.4, \qquad \sum y^2 = 76.8.$$

Dev claims that the mean volume of juice in bottles filled by machine Y is greater than the mean volume of juice in bottles filled by machine X. A test at the α% significance level suggests that there is sufficient evidence to support Dev's claim.

Find the set of possible values of α. [9]

6 A sports college keeps records of the times taken by students to run one lap of a running track. The population median time taken is 51.0 seconds. After a month of intensive training, a random sample of 22 new students run one lap of the track, giving times, in seconds, as follows.

51.3	52.0	53.4	49.2	49.3	51.1	52.2	47.2
53.0	48.5	49.4	50.3	50.8	51.6	49.1	52.3
51.8	52.4	47.9	48.9	50.6	51.9		

It is claimed that the intensive training has led to a decrease in the median time taken to run one lap of the track.

Carry out a Wilcoxon signed-rank test, at the 5% significance level, to test whether there is sufficient evidence to support the claim. [9]

Additional page

If you use the following page to complete the answer to any question, the question number must be clearly shown.

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